Online Appendix to "Monetary Policy, Firm Heterogeneity, and the Distribution of Investment Rates"

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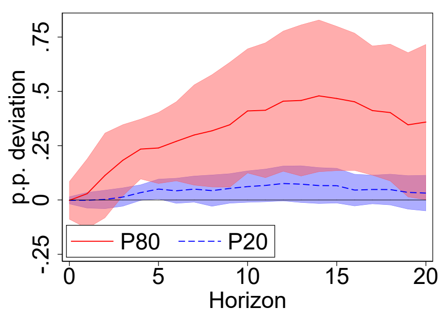
A Additional Tables and Figures

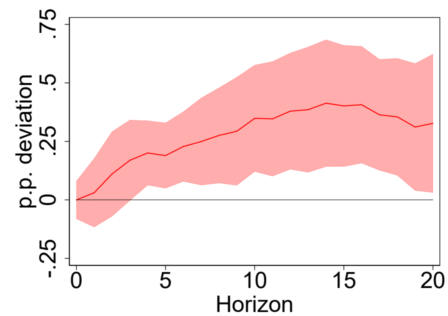
Table A.1: Fixed Parameters

Parameter	Description	Value
Household		
β	Discount factor	0.99
ψ	Labor Disutility	0.58
Investment Block		
θ	Capital Coefficient	0.21
ν	Labor Coefficient	0.64
δ	Depreciation Rate	1.93%
ρ_z	Persistence of Idiosyncratic TFP Shocks	0.95
π^{exit}	Exit Probability	1.63%
New Keynesian Block		
φ	Price Adjustment Cost	90
γ	Elasticity of Substitution over Intermediate Goods	10
φ_π	Taylor Rule Coefficient on Inflation	1.5
ρ_r	Interest Rate Smoothing	0.75
κ	External Capital Adjustment Costs	11

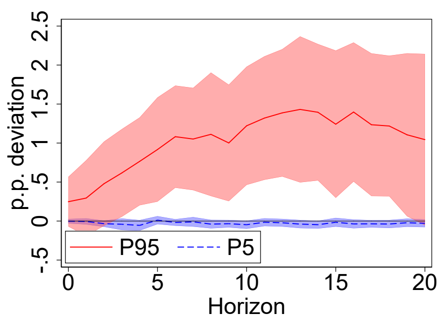
Figure A.1: Effect of Monetary Policy on Quantiles of the Investment Rate Distribution



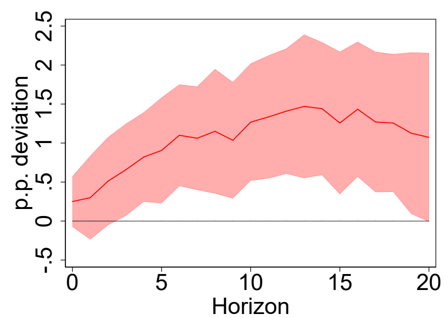
(a) Quantiles (80th & 20th)



(b) Interquantile Range: P80 - P20



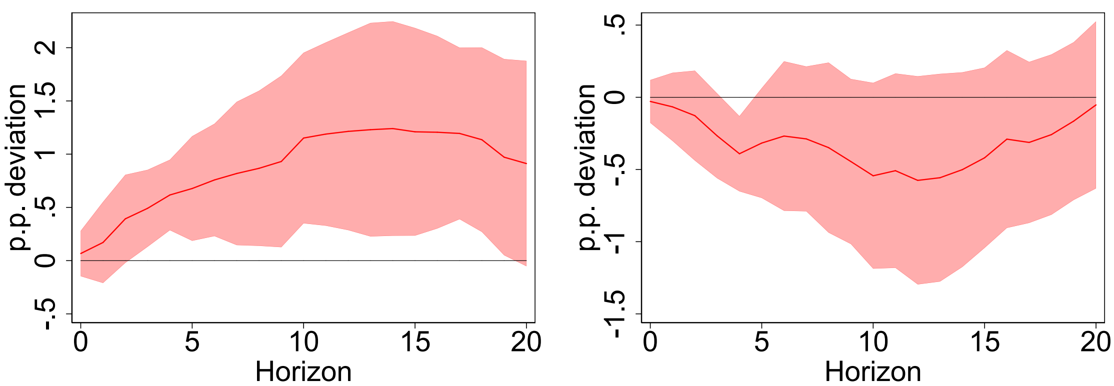
(c) Quantiles (95th & 5th)



(d) Interquantile Range: P95 - P5

Notes: This figure plots the effect of a monetary policy shock on statistics of the investment rate distribution. The lines represent the estimated $\hat{\beta}^h$ from separate regressions: $y_{t+h} - y_{t-1} = \alpha^h + \beta^h \epsilon_t^{MP} + \sum_{j=2}^{4} \gamma^j \mathbb{1}\{q_{t+h} = j\} + e_{t+h}$. The monetary policy shocks are scaled to reduce the 1-year Treasury yield by 25 basis points. The shaded areas indicate the 90% confidence intervals constructed using standard errors that are robust to heteroskedasticity and autocorrelation.

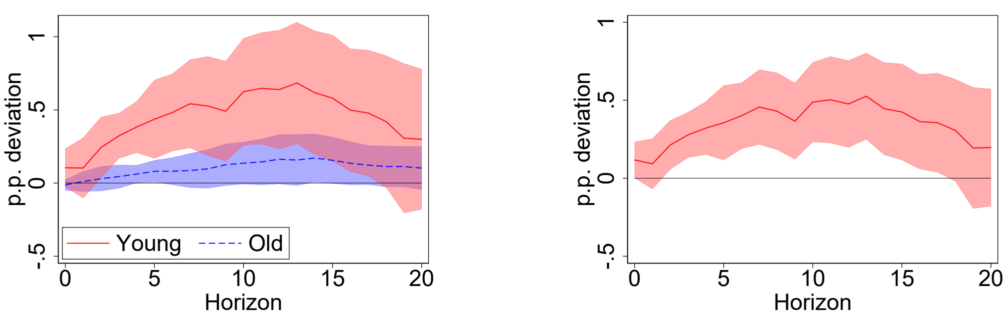
Figure A.2: Effect of Monetary Policy on the Spike and Inaction Rate



(a) Spike Rate **(b)** Inaction Rate

Notes: This figure plots the effect of a monetary policy shock on the spike rate and the inaction rate of all firms. A spike is an investment rate exceeding 10%, inaction is an investment rate less than 0.5% in absolute value. The lines represent the estimated $\hat{\beta}^h$ from separate regressions: $y_{t+h} - y_{t-1} = \alpha^h + \beta^h \epsilon_t^{MP} + \sum_{j=2}^{4} \gamma^j \mathbb{1}\{q_{t+h} = j\} + e_{t+h}$. The monetary policy shocks are scaled to reduce the 1-year Treasury yield by 25 basis points. The shaded areas indicate the 90% confidence intervals constructed using standard errors that are robust to heteroskedasticity and autocorrelation.

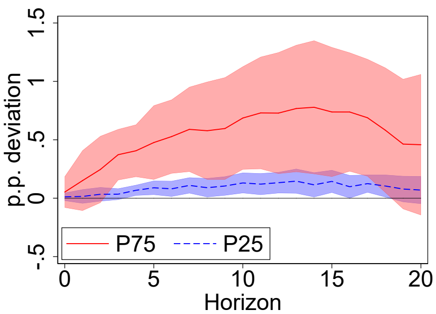
Figure A.3: Effect of Monetary Policy on Age-Group-Specific Average Investment Rates



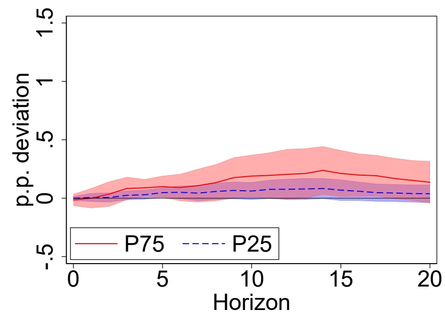
(a) Average Investment Rates **(b)** Difference (Young - Old)

Notes: Young (old) firms are less (more) than 15 years old. The lines represent the estimated $\hat{\beta}^h$ from separate regressions: $y_{t+h} - y_{t-1} = \alpha^h + \beta^h \epsilon_t^{MP} + \sum_{j=2}^{4} \gamma^j \mathbb{1}\{q_{t+h} = j\} + e_{t+h}$. The monetary policy shocks are scaled to reduce the 1-year Treasury yield by 25 basis points. The shaded areas are the 90% confidence intervals constructed using standard errors that are robust to heteroskedasticity and autocorrelation.

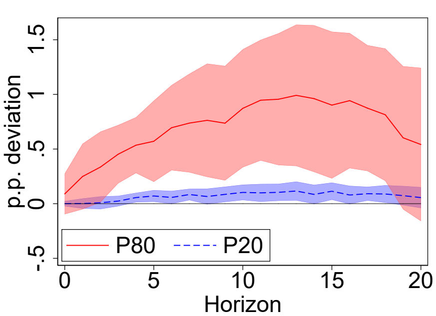
Figure A.4: Effect on Quantiles of Age-Group-Specific Inv. Rate Distributions



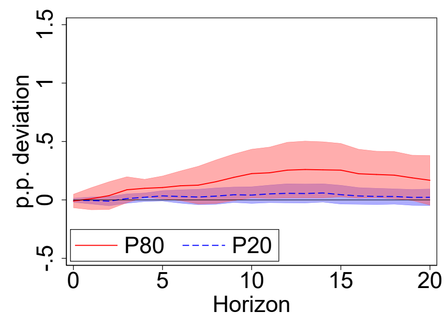
(a) 75th & 25th (Young Firms)



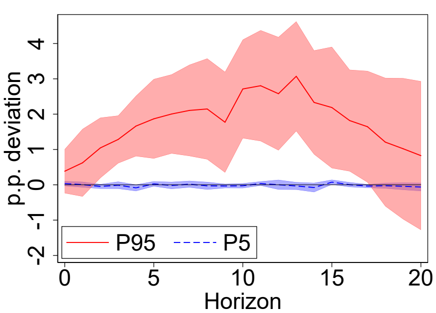
(b) 75th & 25th (Old Firms)



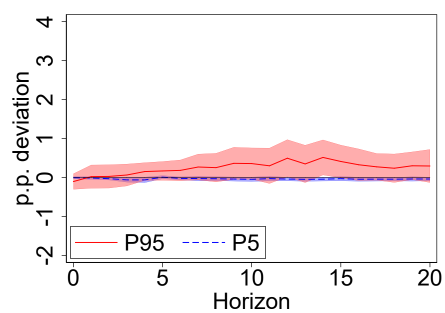
(c) 80th & 20th (Young Firms)



(d) 80th & 20th (Old Firms)



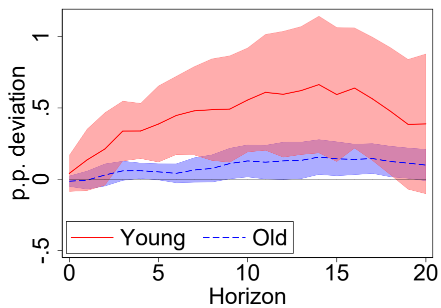
(e) 95th & 5th (Young Firms)



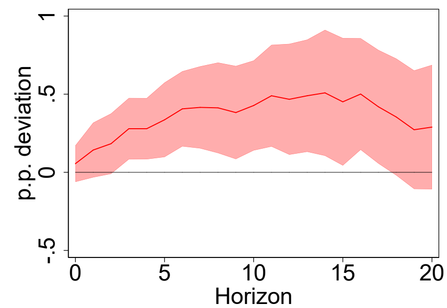
(f) 95th & 5th (Old Firms)

Notes: This figure plots the effect of a monetary policy shock on quantiles of the age-specific investment rate distributions. Young (old) firms are firms less (more) than 15 years old. The lines represent the estimated $\hat{\beta}^h$ from separate regressions: $y_{t+h} - y_{t-1} = \alpha^h + \beta^h \epsilon_t^{MP} + \sum_{j=2}^{4} \gamma^j \mathbb{1}\{q_{t+h} = j\} + e_{t+h}$. The monetary policy shocks are scaled to reduce the 1-year Treasury yield by 25 basis points. The shaded areas indicate the 90% confidence intervals constructed using standard errors that are robust to heteroskedasticity and autocorrelation.

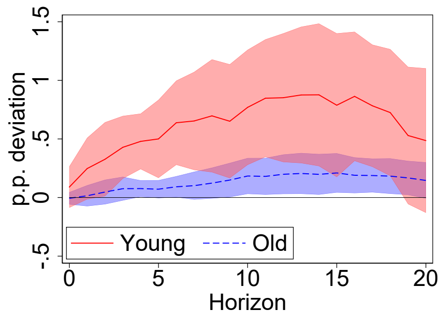
Figure A.5: Effect on Interquantile Ranges of Age-Group-Specific Inv. Rate Distributions



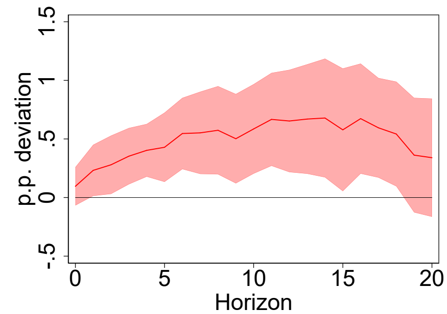
(a) Interquartile Range: P75 - P25



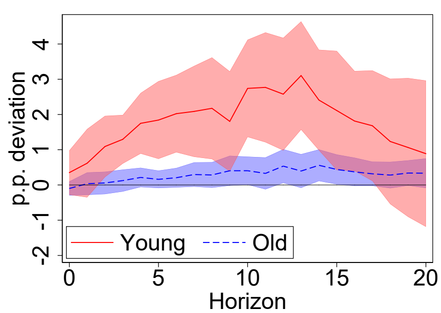
(b) Difference (Young - Old)



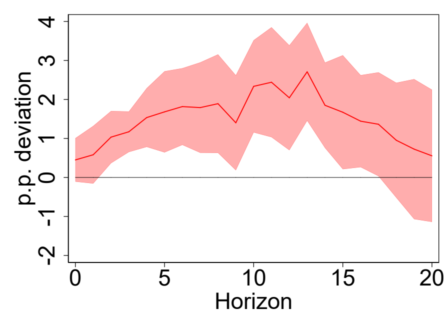
(c) Interquantile Range: P80 - P20



(d) Difference (Young - Old)



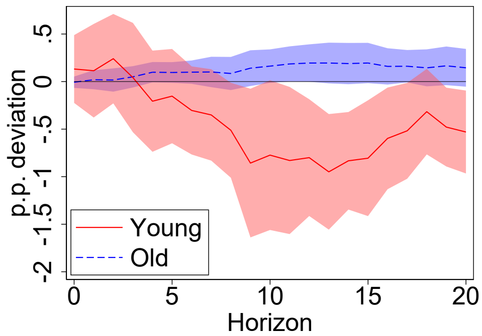
(e) Interquantile Range: P95 - P5



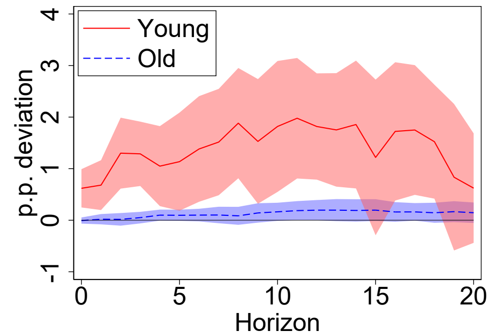
(f) Difference (Young - Old)

Notes: This figure plots the effect of a monetary policy shock on statistics of the age-specific investment rate distributions. Young (old) firms are firms less (more) than 15 years old. The lines represent the estimated $\hat{\beta}^h$ from separate regressions: $y_{t+h} - y_{t-1} = \alpha^h + \beta^h \epsilon_t^{MP} + \sum_{j=2}^{4} \gamma^j \mathbb{1}\{q_{t+h} = j\} + e_{t+h}$. The monetary policy shocks are scaled to reduce the 1-year Treasury yield by 25 basis points. The shaded areas indicate the 90% confidence intervals constructed using standard errors that are robust to heteroskedasticity and autocorrelation.

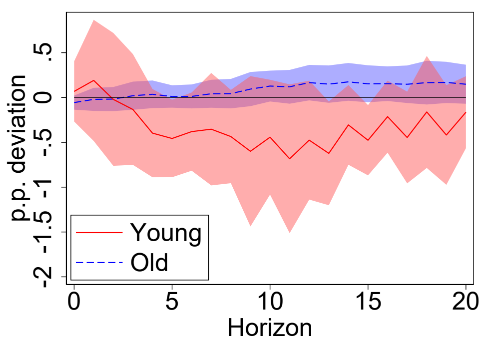
Figure A.6: Effect of Monetary Policy – Low Financial Constraints



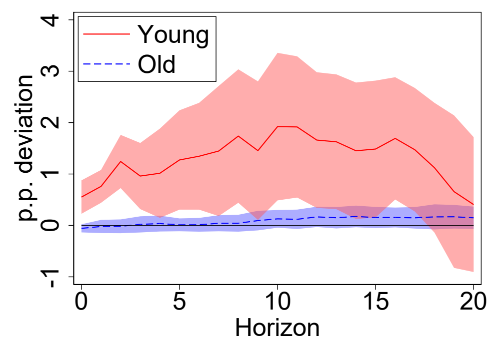
(a) Inaction Rate (Low Leverage)



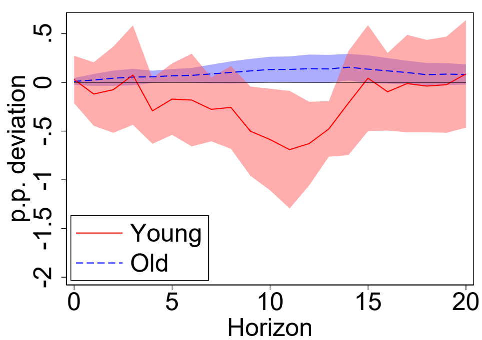
(b) Spike Rate (Low Leverage)



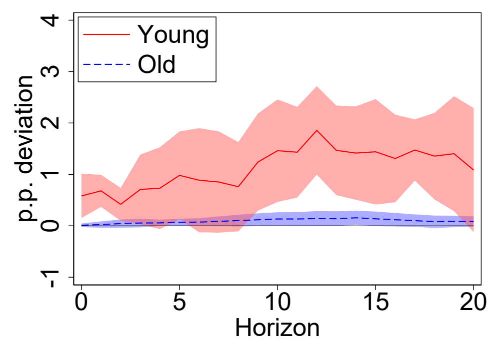
(c) Inaction Rate (High Liquidity)



(d) Spike Rate (High Liquidity)



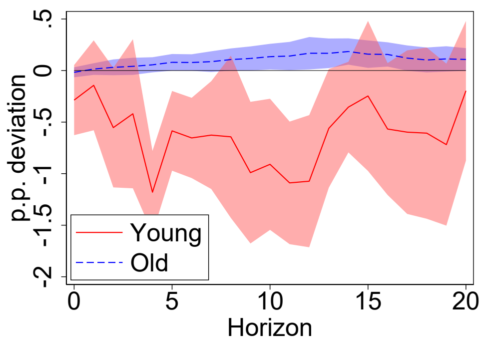
(e) Inaction Rate (Paid Dividends)



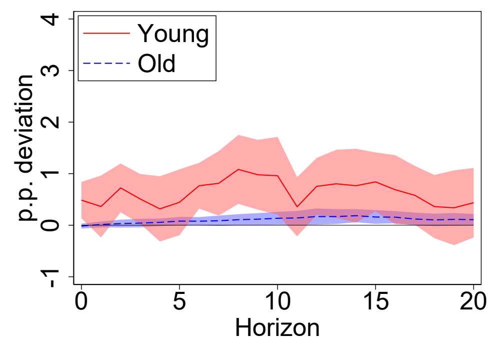
(f) Spike Rate (Paid Dividends)

Notes: This figure plots the effect of a monetary policy shock on the spike rate and the inaction rate of young and old firms. Young (old) firms are less (more) than 15 years old. A spike is an investment rate exceeding 10%, inaction is an investment rate less than 0.5% in absolute value. Panels (a) and (b) only use firm observations which were in the bottom tercile of the leverage distribution in the past quarter, panels (c) and (d) uses observations which were in the top tercile of the liquidity distribution in the past quarter, and panels (e) and (f) use only firm observations which have ever paid dividends in the past. The lines represent the estimated $\hat{\beta^h}$ from separate regressions: $y_{t+h} - y_{t-1} = \alpha^h + \beta^h \epsilon_t^{MP} + \sum_{j=2}^{4} \gamma^j \mathbb{1}\{q_{t+h} = j\}$ $+ e_{t+h}$. The monetary policy shocks are scaled to reduce the 1-year Treasury yield by 25 basis points. The shaded areas are the 90% confidence intervals constructed using standard errors that are robust to heteroskedasticity and autocorrelation.

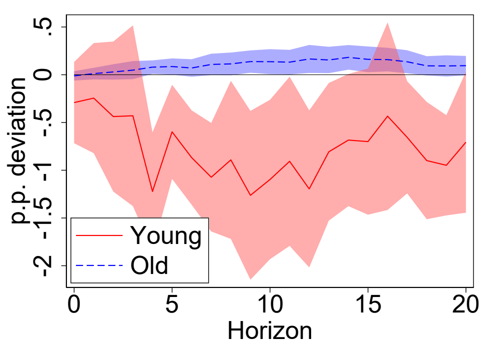
Figure A.7: Effect of Monetary Policy – High Financial Constraints



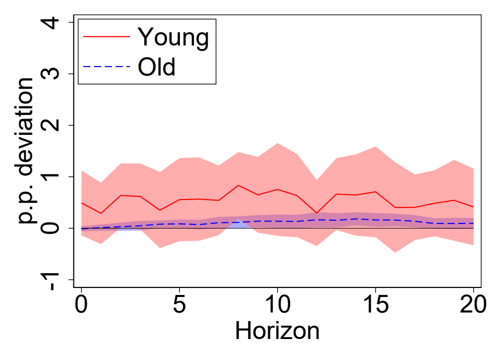
(a) Inaction Rate (High Leverage)



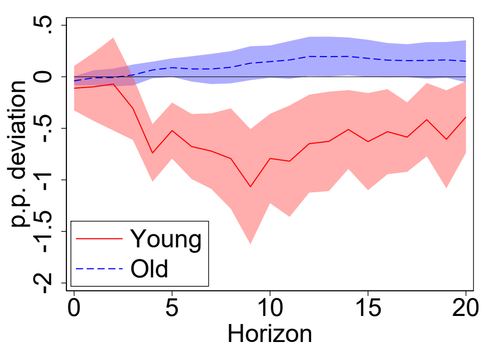
(b) Spike Rate (High Leverage)



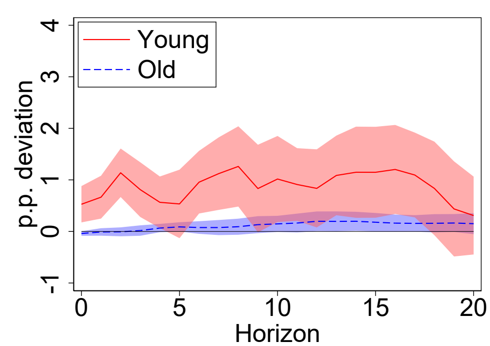
(c) Inaction Rate (Low Liquidity)



(d) Spike Rate (Low Liquidity)



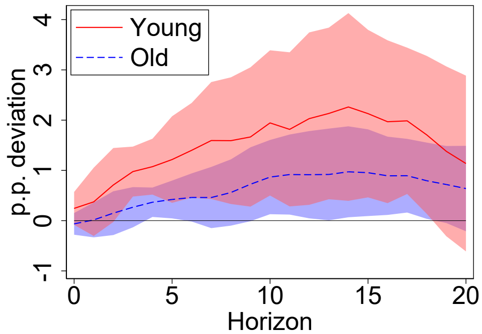
(e) Inaction Rate (Not Paid Dividends)



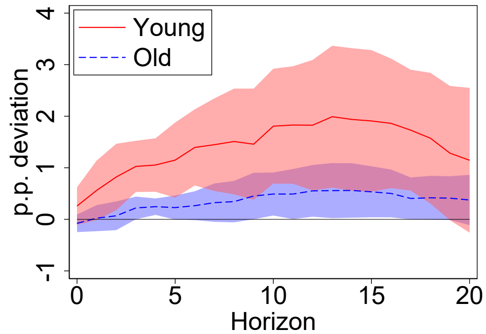
(f) Spike Rate (Not Paid Dividends)

Notes: This figure plots the effect of a monetary policy shock on the spike rate and the inaction rate of young and old firms. Young (old) firms are less (more) than 15 years old. A spike is an investment rate exceeding 10%, inaction is an investment rate less than 0.5% in absolute value. Panels (a) and (b) only use firm observations which were in the top tercile of the leverage distribution in the past quarter, panels (c) and (d) uses observations which were in the bottom tercile of the liquidity distribution in the past quarter, and panels (e) and (f) use only firm observations which have never paid dividends in the past. The lines represent the estimated $\hat{\beta^h}$ from separate regressions: $y_{t+h} - y_{t-1} = \alpha^h + \beta^h \epsilon_t^{MP} + \sum_{j=2}^{4} \gamma^j \mathbb{1}\{q_{t+h} = j\}$ $+ e_{t+h}$. The monetary policy shocks are scaled to reduce the 1-year Treasury yield by 25 basis points. The shaded areas are the 90% confidence intervals constructed using standard errors that are robust to heteroskedasticity and autocorrelation.

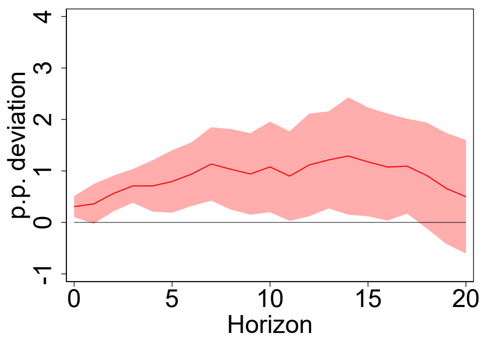
Figure A.8: Alternative Thresholds for Investment Spikes



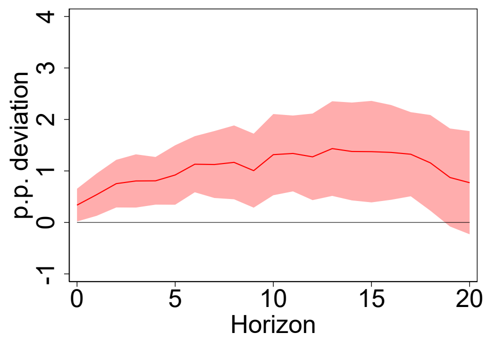
(a) Spike Rate (Threshold: 8%)



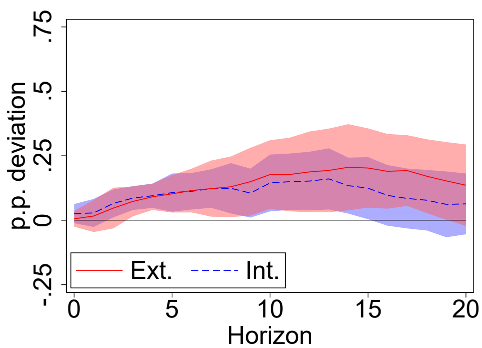
(b) Spike Rate (Threshold: 12%)



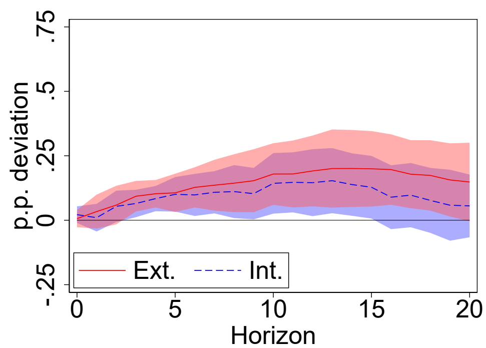
(c) Spike Rate: Young - Old (Threshold: 8%)



(d) Spike Rate: Young - Old (Threshold: 12%)



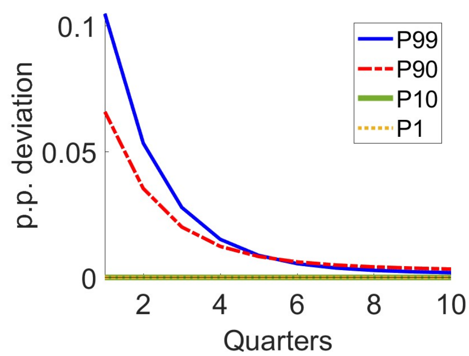
(e) Decomp. Ext. vs. Int. (Threshold: 8%)



(f) Decomp. Ext. vs. Int. (Threshold: 12%)

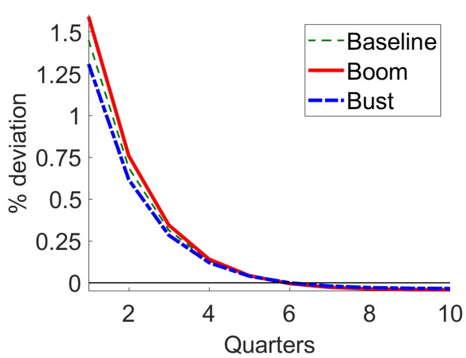
Notes: This figure replicates several main results while using alternative thresholds (8% or 12% vs. 10% in main text) for the definition of an investment spike. Panels (a) and (b) plot the effect of a monetary policy shock on the spike rates of young and old firms (see Figure 3, panel a). Panels (c) and (d) plot the effect on the spike rate difference between young and old firms (see Figure 3, panel b). Panels (e) and (f) plot the decomposition of the effect of a monetary policy shock on the average investment rate (see Figure 4, panel a).

Figure A.9: Effect of Monetary Policy on Quantiles of the Investment Rate Distribution

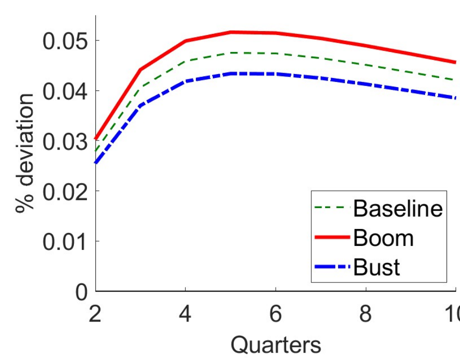


Notes: This figure plots the IRFs of several quantiles of the investment rate distribution in the model.

Figure A.10: State-Dependent Effects of Monetary Policy



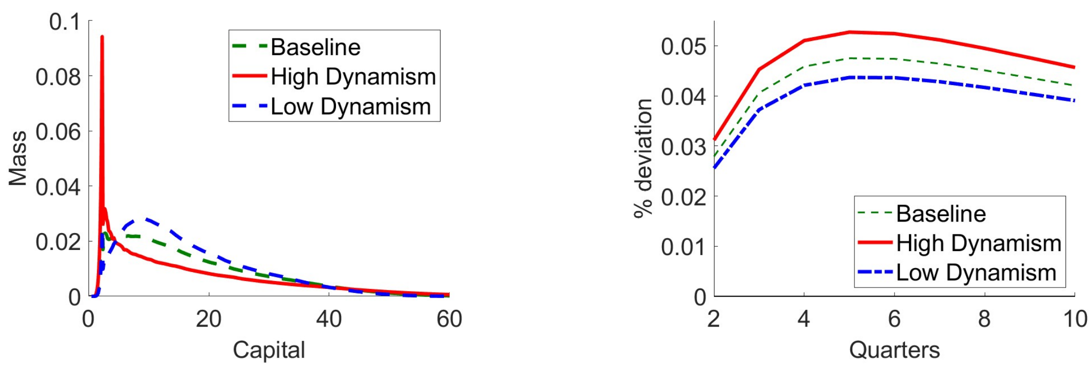
(a) Investment



(b) Capital

Notes: This figure plots the effects of a monetary policy shock on aggregate investment and capital. For the boom (bust) impulse response functions, the monetary policy shock is combined with a TFP shock that increases (decreases) TFP on impact by 5%.

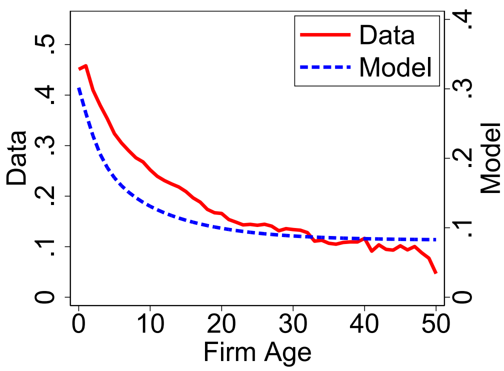
Figure A.11: Firm Aging Affects the Investment Channel of Monetary Policy



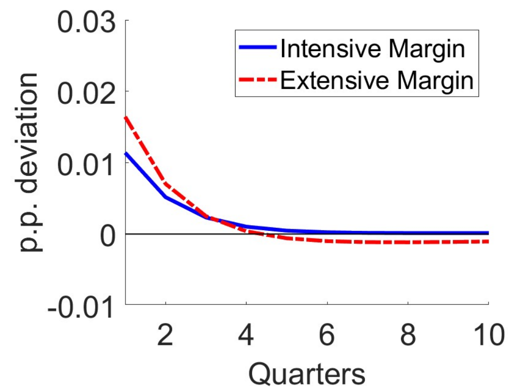
(a) Distribution of Firms (Steady State) **(b)** Effect of Monetary Policy on Capital

Notes: Panel (a) of this figure shows the steady-state distributions of firms over capital. The underlying capital grid is log-spaced. Panel (b) shows the impulse response function of aggregate capital to an expansionary monetary policy shock. The three calibrations only differ with respect to the entry/exit rate which is 6.5% (baseline), 13% (high dynamism), and 3.375% (low dynamism).

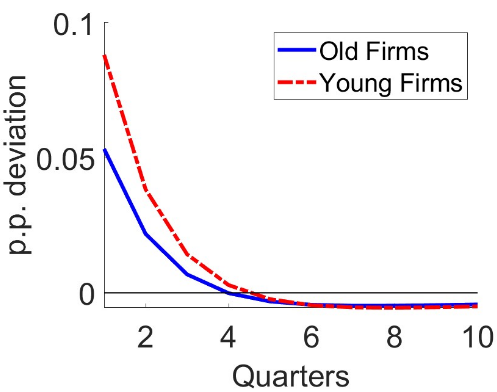
Figure A.12: Model Results using the Spike Rate instead of the Hazard Rate



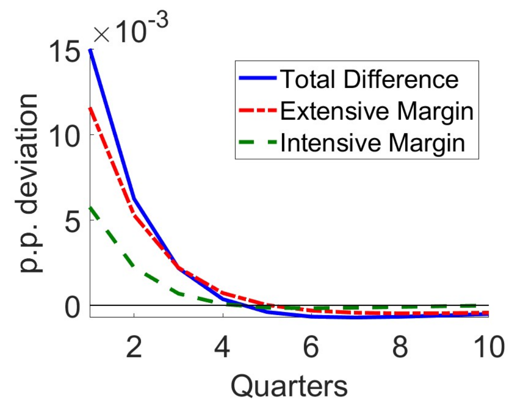
(a) Frequency of Adjustment (Life-Cycle)



(b) Avg. Inv. Rate Decomposition



(c) Spike Rate



(d) Extensive vs. Intensive Margin

Notes: This figure reproduces several model results using the spike rate instead of the hazard rate to identify extensive margin adjustments. Panel (a) plots the life-cycle profiles of the spike rate in the model and in the data (see Figure 5, panel (b)). Panel (b) decomposes the IRF of the average investment rate into contributions of the intensive and extensive margins (see Figure 7, panel b). Panels (c) and (d) plot the IRFs of the spike rate and the decomposition of the heterogeneous effect into intensive and extensive margins (see Figure 9, panels (b) and (c)).

B Simple Model

In the main text, we build a heterogeneous-firm life-cycle model with capital adjustment costs and nominal rigidities. The purpose is to explain the observed (heterogeneous) effects of interest rate changes on the distribution of investment rates and study their aggregate implications. In the current section, we illustrate the mechanisms at work through the lens of a simple two-period model. Most importantly, the model features fixed capital adjustment costs which create an extensive margin investment decision.

In this simple model, we compare small and large firms. Since age and size are strongly correlated both in the data and in the quantitative model, all intuitions we provide in the simple model hold true when comparing young and old firms in the quantitative model. In Appendix C, we compare the heterogeneous sensitivity by age and by size in the data and in the quantitative model.

The simple model consists of two periods. In period 0, firms are endowed with k_0 units of capital and choose the next period's capital k_1. The price of one unit of capital relative to the price of the consumption good is q. In period 1, firms transform capital into the consumption good (y) using the decreasing returns to scale production technology $y = k_1^\theta$ with $\theta < 1$. Sales are discounted at the real interest rate r, and capital depreciates fully during production.

In the absence of adjustment costs, the firms' profit-maximization problem is

$$\max_{k_1} \frac{1}{1+r} k_1^\theta - q(k_1 - k_0). \tag{1}$$

From the first-order condition for k_1, we obtain the optimal amount of capital that the firm chooses for period 1

$$k_1^* = \left(\frac{\theta}{(1+r)q} \right)^{\frac{1}{1-\theta}} \tag{2}$$

and the optimal (gross) investment rate as a function of firm size $i^*(k_0) = \frac{k_1^*}{k_0}$.

We now introduce some features from the quantitative model. First, there is a unit mass of firms within each size category k_0 and firms are indexed by j. Second, adjusting the stock of capital is subject to a fixed adjustment cost $\xi_j \in [0, \overline{\xi}]$, which is drawn from a uniform distribution. Moreover, we assume that the economy is populated by firms whose initial capital stocks are below the desired level, i.e., $k_{j,0} < k_1^*$, $\forall k_0$.[1]

[1] In the steady state of the quantitative model, there are also some firms with capital stocks above their

The optimization problem of a firm j with an initial stock of capital k_0 has changed to:

$$\max_{k_{1,j}} \frac{1}{1+r} k_{1,j}^{\theta} - q(k_{1,j} - k_0) - \xi_j \mathbb{1}\{k_{1,j} \neq k_0\}, \tag{3}$$

where $\mathbb{1}\{k_{1,j} \neq k_0\}$ is an indicator variable that equals 1 if $k_{1,j} \neq k_0$ and 0 otherwise. To solve this problem, let $VA(k_0)$ denote the value added of adjusting capital while ignoring the fixed adjustment cost:

$$VA(k_0) = \frac{1}{1+r} k_1^{*\theta} - q(k_1^* - k_0) - \frac{1}{1+r} k_0^{\theta}, \tag{4}$$

where k_1^* is the optimal amount of capital that firms will acquire conditional on adjusting as defined by equation (2).

Considering the adjustment cost, a firm j adjusts capital if and only if the value added exceeds the costs, i.e., $VA(k_0) > \xi_j$. The threshold value of ξ_j, which makes a firm indifferent between adjusting or not, is defined by $\xi^T(k_0) \equiv VA(k_0)$. This implies a cutoff rule, i.e., a firm j will adjust its capital stock if and only if $\xi_j < \xi^T(k_0)$. From equation (4), it is evident that this cutoff value not only depends on the initial size of the firm but also on the interest rate r and the other parameters of the model.

The average investment rate among firms of size k_0 is:

$$\bar{i}(k_0) = \lambda(k_0) \times i^*(k_0) \tag{5}$$

where $\lambda(k_0) = \frac{\xi^T(k_0)}{\bar{\xi}} \in [0,1]$ denotes the share of firms of size k_0 that choose to invest, i.e. the hazard rate. Conditional on investing, firms choose the optimal investment rate $i^*(k_0)$ as defined above.

The group-specific interest rate sensitivity of the investment rate is:

$$\frac{\partial \bar{i}(k_0)}{\partial r} = \underbrace{\frac{\partial \lambda(k_0)}{\partial r} i^*(k_0)}_{\text{Extensive Margin}} + \underbrace{\lambda(k_0) \frac{i^*(k_0)}{\partial r}}_{\text{Intensive Margin}}, \tag{6}$$

which features two components. There is an intensive margin effect, $\lambda(k_0)\frac{i^*(k_0)}{\partial r}$, because firms that would be adjusting anyways choose a different investment rate. Moreover, there is an extensive margin effect, $\frac{\partial \lambda(k_0)}{\partial r} i^*(k_0)$, because more or less firms choose to invest

desired level. However, quantitatively, these firms play a minor role.

at all. Motivated by our empirical findings, this paper emphasizes the extensive margin effect.

Proposition 1 provides the main theoretical findings of this paper, which regard the effect of interest rate changes on the hazard rate ($\frac{\partial \lambda(k_0)}{\partial r}$) as well as how the sensitivity of the average investment rate due to the extensive margin changes with firm size.

Proposition 1. *In an economy populated by heterogeneous-firms that face fixed adjustment costs as described above, it holds that*

1. *An interest rate cut increases the hazard rate:* $\frac{\partial \lambda(k_0)}{\partial r} < 0$
2. *The sensitivity of the average investment rate to interest rate changes via the extensive margin is decreasing (in absolute terms) in firm size:* $\frac{\partial \left(\frac{\partial \lambda(k_0)}{\partial r} i^*(k_0) \right)}{\partial k_0} > 0$

Proof. See Appendix B.1. □

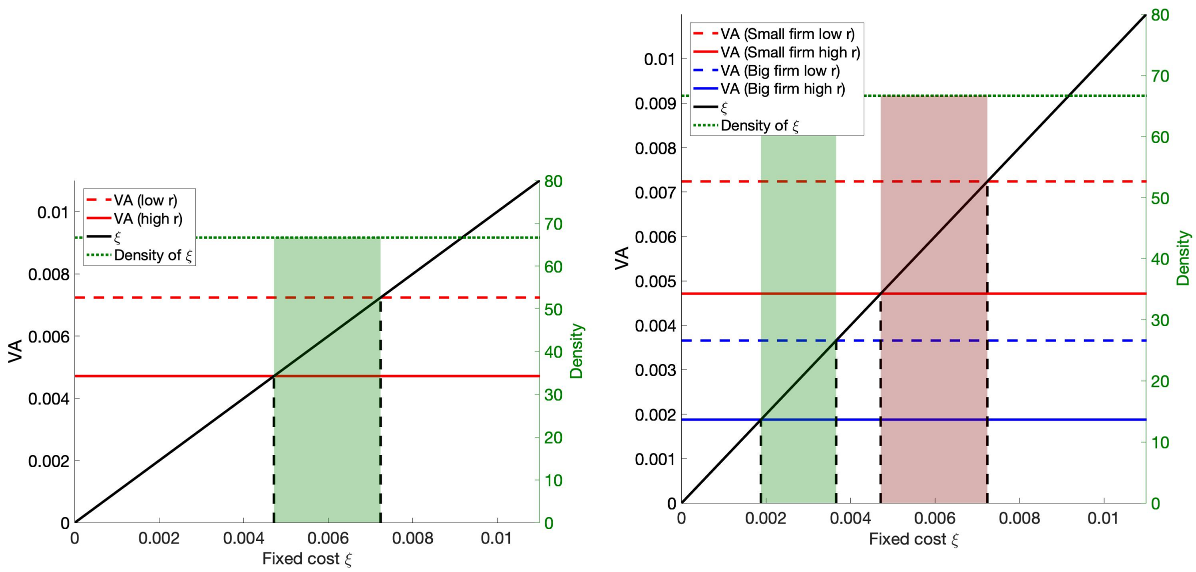
The first part of Proposition 1 establishes that an interest rate cut increases the hazard rate in line with the empirical evidence shown in Figure 3. The costs of investing (cost of additional capital, adjustment cost) are paid in period 0, whereas the benefits materialize in period 1. When the interest rate falls, the discounted benefit of investing rises. Hence, the value added of adjusting and thus the hazard rate rise.[2]

Figure B.1a provides visual intuition by plotting the value added for a given k_0, $VA(k_0)$, against the random fixed cost ξ. The black upward-sloping line is the 45° line indicating the points where VA equals ξ. The intercept of the two curves pins down the cutoff value ξ^T. The green dotted line plots the density function of ξ (uniform distribution). The area under the density function to the left of the cutoff value ξ^T is the mass of adjusting firms. An interest rate cut shifts the VA curve upwards. As a result, the cutoff value ξ^T increases and so does the mass of adjusting firms as indicated by the green shaded area.

The second part of Proposition 1 establishes that the effect of an interest rate cut on the group-specific average investment rate via the extensive margin is larger among small firms. To understand this result, it is useful to compare the extensive margin effect for

[2]In the quantitative model, there are of course additional effects, but the main intuition – an interest rate cut raising the value added of investing – remains the same.

Figure B.1: Intuition for Proposition 1



(a) r-Sensitivity of Hazard Rate **(b) r-Sensitivity by Firm Size**

Notes: This figure plots the value added of investing (VA) of a firm against the random fixed cost ξ. The black upward-sloping line is the 45° line indicating the points where VA equals ξ. The intercept of the two curves pins down the threshold value of ξ^T. The green dotted line plots the density function of ξ (uniform distribution). The area under the density function to the left of the threshold value ξ^T is the hazard rate. The shaded area in Panel (a) plots the difference in the hazard rate after an interest rate change. Panel (b) plots the difference in the hazard rate for a small and a big firm.

groups of small (S) and large (L) firms:

$$\begin{aligned} HetExt_{S-L} &= \underbrace{\frac{\partial\lambda(k_{0,S})}{\partial r} i^*(k_{0,S})}_{\text{Small Firms}} - \underbrace{\frac{\partial\lambda(k_{0,L})}{\partial r} i^*(k_{0,L})}_{\text{Large Firms}} \\ &= \underbrace{\frac{\partial\lambda(k_{0,L})}{\partial r}\left(i^*(k_{0,S}) - i^*(k_{0,L})\right)}_{\text{Heterogeneous Size Effect}} + \underbrace{\left(\frac{\partial\lambda(k_{0,S})}{\partial r} - \frac{\partial\lambda(k_{0,L})}{\partial r}\right) i^*(k_{0,S})}_{\text{Heterogeneous Hazard Rate Increase}} \end{aligned} \tag{7}$$

This decomposition shows that there are two mechanisms. First, there is the *heterogeneous size effect*, due to which even if an interest rate cut had the same effect on hazard rates of small and large firms, there would be a differential effect on average investment rates. This is because among the *new* adjusters, small firms have higher investment rates conditional on adjusting ($i^*(k_{0,S}) - i^*(k_{0,L}) > 0$). This follows from the observation that in this simple model, conditional on investing, all firms choose k_1^* and the investment rate is defined by $i^* = \frac{k_1^*}{k_0}$. In the absence of an extensive margin investment decision, this effect would disappear because $\frac{\partial \lambda(k_0)}{\partial r} = 0$.

Second, interestingly, an interest rate cut increases the hazard rate of small firms by more than the hazard rate of large firms. This result aligns well with the empirical evidence that the spike rate of small (young) firms reacts more strongly to a monetary shock than the spike rate of large (old) firms (see Figure C.2 for size and Figure 3 for age). As discussed above, the hazard rate rises, because the value added of investing rises, which happens because the discounted benefit of investing rises. This increase in the discounted benefit of investing is larger for small firms. The reason for this is that small firms have a higher marginal product of capital because of decreasing returns to scale. Hence, the interest rate cut has a larger effect on the hazard rate of small firms.

Figure B.1b provides visual intuition for the heterogeneous effect of an interest rate cut on hazard rates. The cut in the interest rate shifts the VA of small firms (red lines) up by more than the VA of big firms (blue lines). As a result, the change in the hazard rate is more pronounced for small firms (red-shaded area) than for big firms (green-shaded area).

To sum up, we have highlighted two effects in this simple model. First, an interest rate cut increases the hazard rate, i.e. the fraction of firms deciding to make an investment. Therefore, a change in the interest rate changes the distribution of investment rates. Second, the average investment rate of small firms responds more strongly along the extensive margin to interest rate changes than the average investment rate of large firms.

Regarding the second effect, it is worth pointing out that small firms are more sensitive to interest rate changes *in the absence* of a financial accelerator mechanism. The basic idea of the financial accelerator mechanism is that interest rate changes affect financing conditions and small firms are more exposed to financing conditions than large firms. Then, interest rate changes have a heterogeneous effect on investment because there is a heterogeneous effect on the *cost* of investing, as e.g. in Ottonello and Winberry (2020). In contrast, in this model, there is a heterogeneous effect of interest rate changes on investment because of a heterogeneous effect on the *benefit* of investing.[3] This is because small firms have a higher marginal product of capital.

[3]Even though the capital adjustment costs that we impose can in principle be interpreted as stand-ins for financial frictions, the model does not feature a financial accelerator mechanism. This is because by construction, the capital adjustment costs are themselves not affected by aggregate shocks, including monetary policy shocks.

B.1 Proofs

Proposition 1. *In an economy populated by heterogeneous-firms that face fixed adjustment costs as described above, it holds that*

1. *An interest rate cut increases the hazard rate:* $\frac{\partial \lambda(k_0)}{\partial r} < 0$
2. *The sensitivity of the average investment rate to interest rate changes via the extensive margin is decreasing (in absolute terms) in firm size:* $\frac{\partial \left(\frac{\partial \lambda(k_0)}{\partial r} i^*(k_0) \right)}{\partial k_0} > 0$

Proof. Rearranging equation (4), the value added of adjusting capital while ignoring the fixed adjustment cost is:

$$VA(k_0) = \frac{1}{1+r} \left(k_1^{*\theta} - k_0^{\theta} \right) - q(k_1^* - k_0) \tag{8}$$

where k_1^* was defined in equation (2). Using the definition of the cutoff $\xi^T(k_0)$ and the hazard rate $\lambda(k_0)$ from the main text, we have

$$\lambda(k_0) = \frac{1}{\bar{\bar{\xi}}} VA(k_0). \tag{9}$$

Taking the derivative w.r.t. the real interest rate, we get

$$\frac{\partial \lambda(k_0)}{\partial r} = -\frac{1}{\bar{\bar{\xi}}} \frac{1}{(1+r)^2} \left(k_1^{*\theta} - k_0^{\theta} \right) < 0, \tag{10}$$

which proves the first part of the proposition. Note that $k_0 < k_1^*$ by assumption.

The second part of the proposition requires

$$\frac{\partial \left(\frac{\partial \lambda(k_0)}{\partial r} i^*(k_0) \right)}{\partial k_0} = \frac{\partial^2 \lambda(k_0)}{\partial r \partial k_0} i^*(k_0) + \frac{\partial \lambda(k_0)}{\partial r} \frac{\partial i^*(k_0)}{\partial k_0} > 0. \tag{11}$$

The first term is positive, because

$$\frac{\partial^2 \lambda(k_0)}{\partial r \partial k_0} = \frac{1}{\bar{\bar{\xi}}} \frac{1}{(1+r)^2} \theta k_0^{\theta - 1} > 0 \tag{12}$$

and $i^*(k_0) > 0$ because $k_0, k_1 > 0$. The second term is positive because

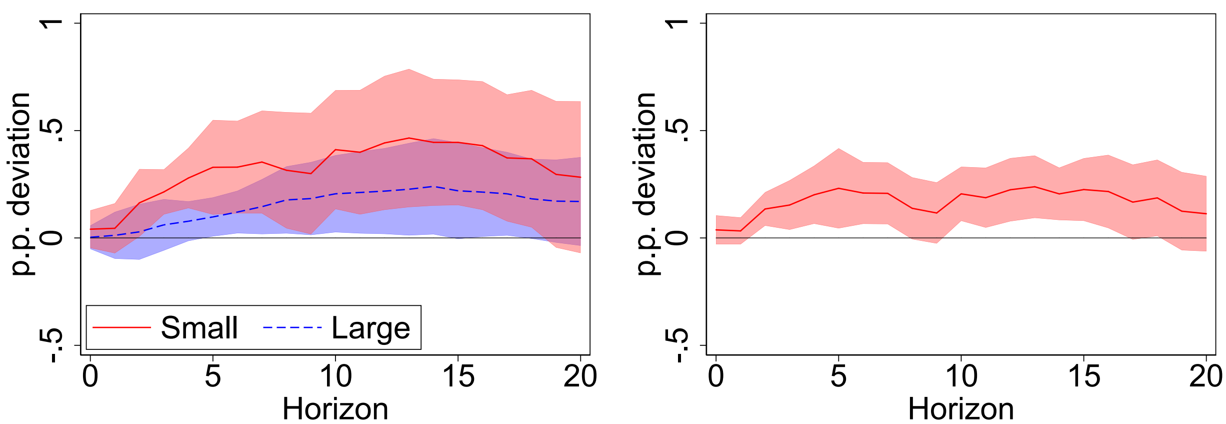
$$\frac{\partial i^*(k_0)}{\partial k_0} = -k_1^* k_0^{-2} < 0 \tag{13}$$

and $\frac{\partial \lambda(k_0)}{\partial r} < 0$ as shown in equation (10). Thus, the inequality in equation (11) holds which completes the proof. $\square$

C Heterogeneous Effects by Firm Size

Empirical Evidence Cloyne et al. (2023) have shown that being young is a better predictor of a firm's sensitivity to monetary policy shocks than being small. We replicate this finding in Figure C.1. Firms that are smaller than the median are at the peak on average 24 basis points more sensitive than firms which are larger than the median. In comparison, young firms are at the peak on average 53 basis points more sensitive than old firms, as shown in Figure A.3. This weaker heterogeneous effect goes along with a weaker heterogeneous effect along the extensive margin, as shown in Figure C.2, which replicates Figure 3 while grouping firms by size instead of age. In addition, the change in the distribution differs somewhat less across size groups than across age groups, as can be seen from comparing Figure A.4 with Figure C.4.

Figure C.1: Effect of Monetary Policy Shock on Average Investment Rates by Size



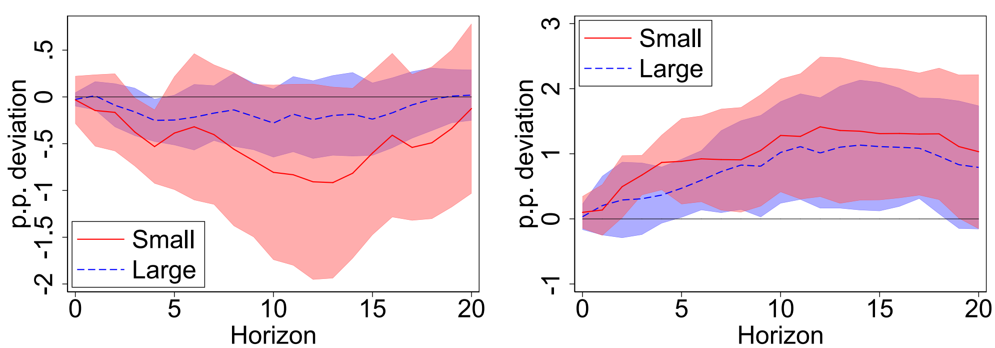
(a) Average Investment Rates

(b) Difference (Small - Large)

Notes: Small (large) firms are firms smaller (larger) than the median in a given quarter. The lines represent the estimated $\hat{\beta}^h$ from separate regressions: $y_{t+h} - y_{t-1} = \alpha^h + \beta^h \epsilon_t^{MP} + \sum_{j=2}^{4} \gamma^j \mathbb{1}\{q_{t+h} = j\} + e_{t+h}$. The monetary policy shocks are scaled to reduce the 1-year Treasury yield by 25 basis points. The shaded areas are the 90% confidence intervals constructed using standard errors that are robust to heteroskedasticity and autocorrelation.

Model Predictions Our model is able to replicate the finding that young age is a better predictor of firms' sensitivity to monetary policy shocks than small size. This is evident from Figure C.3, which replicates Figure 9, panel (a), while grouping firms by size instead of age. Firms that are smaller than the median are on impact more sensitive than firms larger than the median, but the difference is substantially smaller than the gap between young and old firms. Intuitively, age is the better predictor of sensitivity, because young firms are more likely to be "close to making a large investment". This is because young firms are born small and will almost certainly grow in the future. In contrast, small firms may or may not be "close to making a large investment". This is because some firms are small because they are very unproductive, such that the low level of capital is their desired level of capital. In a nutshell, size correlates positively with productivity, while age is uncorrelated with productivity in our model.

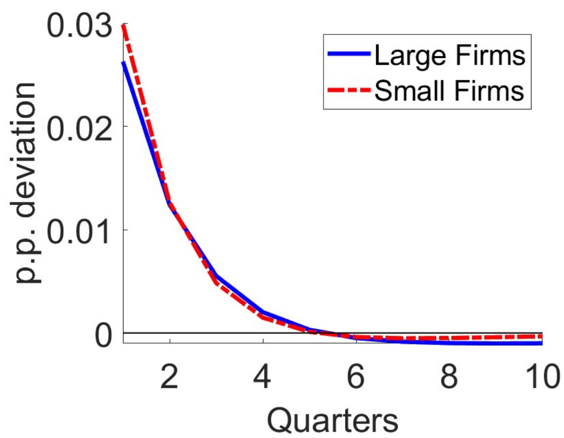
Figure C.2: Effect on Group-Specific Spike & Inaction Rates (by Size)



(a) Spike Rate (by Size) **(b)** Inaction Rate (by Size)

Notes: This figure plots the effect of a monetary policy shock on the spike rate and the inaction rate of small and large firms. Small (large) firms are firms smaller (larger) than the median in a given quarter. A spike rate is an investment rate exceeding 10%, an inaction rate is an investment rate less than 0.5% in absolute value. The lines represent the estimated $\hat{\beta}^h$ from separate regressions: $y_{t+h} - y_{t-1} = \alpha^h + \beta^h \epsilon_t^{MP} + \sum_{j=2}^{4} \gamma^j \mathbb{1}\{q_{t+h} = j\} + e_{t+h}$. The monetary policy shocks are scaled to reduce the 1-year Treasury yield by 25 basis points. The shaded areas are the 90% confidence intervals constructed using standard errors that are robust to heteroskedasticity and autocorrelation.

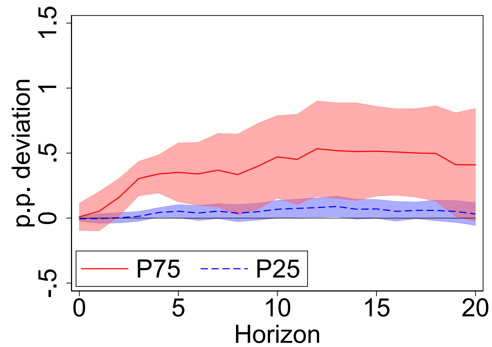
Figure C.3: Heterogeneous Effect (by Size Group) of an Exp. Monetary Policy Shock



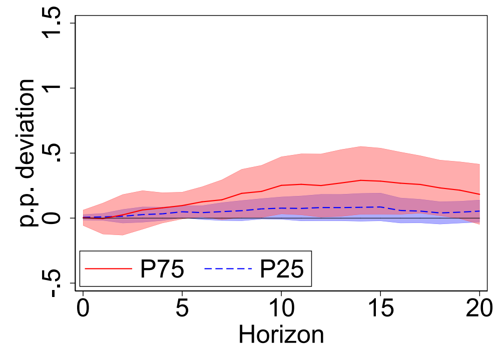
(a) Avg. Investment Rate

Notes: This figure plots the effect of a monetary policy shock on the average investment rates of small and large firms in the model. Small (large) firms are firms smaller (larger) than the median in a given quarter.

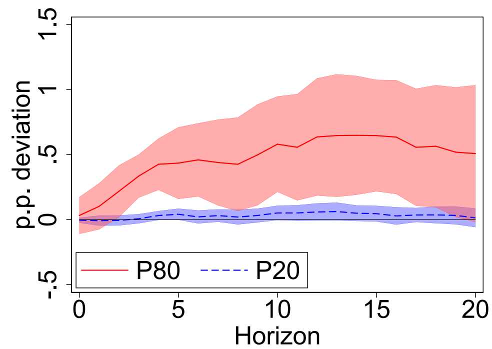
Figure C.4: Effect on Quantiles of Size-Group-Specific Inv. Rate Distributions



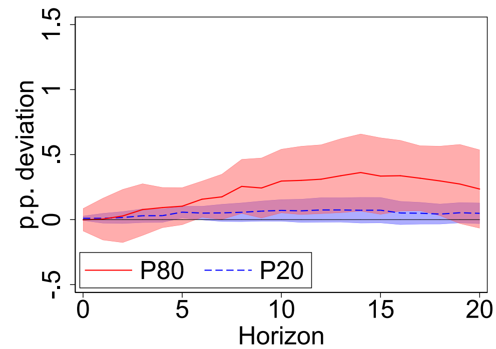
(a) 75th & 25th (Small Firms)



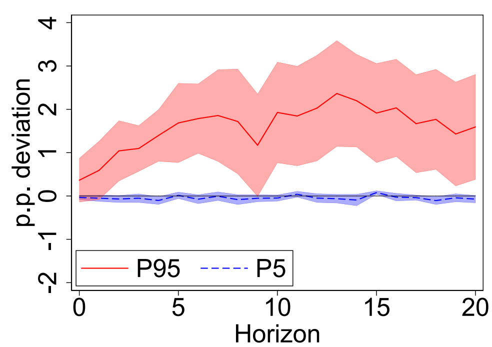
(b) 75th & 25th (Large Firms)



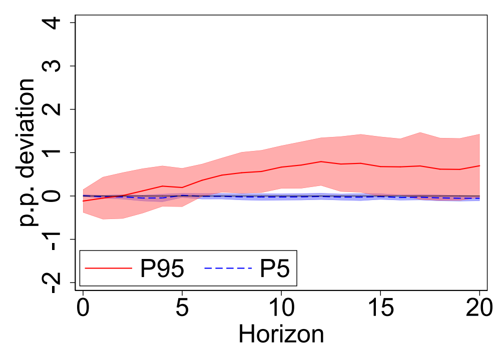
(c) 80th & 20th (Small Firms)



(d) 80th & 20th (Large Firms)



(e) 95th & 5th (Small Firms)



(f) 95th & 5th (Large Firms)

Notes: This figure plots the effect of a monetary policy shock on quantiles of the size-specific investment rate distributions. Small (large) firms are firms smaller (larger) than the median in a given quarter. The lines represent the estimated $\hat{\beta^h}$ from separate regressions: $y_{t+h} - y_{t-1} = \alpha^h + \beta^h \epsilon_t^{MP} + \sum_{j=2}^{4} \gamma^j \mathbb{1}\{q_{t+h} = j\}$ $+ e_{t+h}$. The monetary policy shocks are scaled to reduce the 1-year Treasury yield by 25 basis points. The shaded areas indicate the 90% confidence intervals constructed using standard errors that are robust to heteroskedasticity and autocorrelation.

D Data Appendix

D.1 Sample Selection

We use the Compustat North America Fundamentals Quarterly database. Observations are uniquely identified by GVKEY & DATADATE. In line with the literature, we exclude observations which fall under the following criteria

1. not incorporated in the United States (based on FIC)
2. native currency not U.S. Dollar (based on CURNCDQ)
3. fiscal quarter does not match calendar quarter (based on FYR)
4. specific sectors
   - Utilities (SIC 4900-4999)
   - Financial Industry (SIC 6000-6999)
   - Non-operating Establishments (SIC 9995)
   - Industrial Conglomerates (SIC 9997)
   - Non-classifiable (NAICS > 999900)
5. missing industry information (SIC or NAICS code)
6. missing capital expenditures (based on CAPX)
7. missing or non-positive total assets (AT) or net capital (PPENT)
8. negative sales (SALEQ)
9. acquisitions (based on AQCY) exceed 5% of total assets (in absolute terms)
10. missing or implausible age information (see Appendix D.2)
11. outlier in the Perpetual Inventory Method (see Appendix D.3)

Our sample begins with 1986Q1 and ends with 2018Q4. In a final step, we exclude firm which we observe for less than 20 quarters, unless they are still in the sample in the final period. This ensures that we do not mechanically exclude all firms incorporated in the last five years of our sample.

D.2 Firm Age

We use data on firm age from WorldScope and Jay Ritter's database[4]. WorldScope provides the date of incorporation (Variable: INCORPDATE), while Jay Ritter's database provides the founding date. Both are merged with Compustat based on CUSIP. We define as the firm entry quarter the minimum of both dates if both are available. We do not use information on the initial public offering (IPO) of a firm to determine its age, since the time between incorporation and IPO can vary substantially. However, we use the IPO date to detect implausible age information. We exclude firms for which the IPO date reported in Compustat (IPODATE) precedes the firm entry quarter by more than four quarters. In similar fashion, we exclude firms which appear in Compustat more than four quarters before the firm entry quarter.[5] Finally, we merge information on the beginning of trading from CRSP (Variable: BEGDAT) based on CUSIP and likewise exclude firms with trading more than four quarters before the firm entry quarter.

D.3 Perpetual Inventory Method

Accounting capital stocks $k^a_{j,t}$ as reported in Compustat deviate from *economic* capital stocks for at least two reasons. First, accounting depreciation is driven by tax incentives and usually exceeds economic depreciation. Second, accounting capital stocks are reported at historical prices, not current prices. With positive inflation, both issues make the economic capital stock exceed the accounting capital stock. Therefore, we use a Perpetual Inventory Method (PIM) to compute real economic capital stocks, building on Bachmann and Bayer (2014).

Investment. In principle, there are two options to measure net nominal quarterly investment. First, investment can be measured directly ($I^{dir}_{j,t}$) from the Statement of Cash Flows as capital expenditures (CAPX) less the sale of PPE (SPPE)[6]. Second, investment can be backed out ($I^{indir}_{j,t}$) from the change in PPE (D.PPENT) plus depreciation (DPQ), using Balance Sheet and Income Statement information. Either measure needs to be deflated to obtain real investment. We use INVDEF from FRED, which has the advantage of being quality-adjusted. We prefer the direct investment measure, since the indirect

[4]https://site.warrington.ufl.edu/ritter/

[5]We do not construct firm age from the first appearance in Compustat. An inspection of the data reveals that this would result in wrongly classifying a number of old and established firms as young. Cloyne et al. (2023) do so but show in an earlier working paper version that results are unchanged if only age information from WorldScope is used.

[6]We follow Belo et al. (2014) and set missing values of SPPE to zero.

measure basically captures any change to PPE, including changes due to acquisitions. Nevertheless, we want to exclude observations where both investment measures differ strongly. To this end, we compute investment rates using lagged net accounting capital (L.PPENT), compute the absolute difference between both and discard the top 1% of that distribution.

Depreciation Rates. We obtain economic depreciation rates from the Bureau of Economic Analysis' (BEA) Fixed Asset Accounts. Specifically, we retrieve current-cost net stock and depreciation of private fixed assets by year and industry.[7] We calculate annual depreciation rates by industry and assume a constant depreciation rate within the calendar year to calculate quarterly depreciation rates.

Real Economic Capital Stocks. We initialize a firm's capital stock with the net (real) accounting capital stock $k^a_{j,1}$ (PPENT / INVDEF) whenever this variable is first observed. We iterate forward using deflated investment and the economic depreciation rate.

$$k^{(1)}_{j,1} = k^a_{j,1} \tag{14}$$

$$k^{(1)}_{j,t+1} = (1-\delta^e_t)k^{(1)}_{j,t} + \frac{p^I_t}{p_{2009,t}} I^{dir}_{j,t} \tag{15}$$

Comparing $k^{(1)}_{j,t}$ and $k^a_{j,t}$ shows non-negligible discrepancies. On average, the economic capital stock is larger, confirming the hypothesis that accounting capital stocks are understated. This makes it problematic to use the accounting capital stock as a starting value in the PIM. As a remedy, we again follow Bachmann and Bayer (2014) and use an iterative procedure to re-scale the starting value. We compute a time-invariant scaling factor ϕ at the sector-level and use it to re-scale the starting value as follows. We iterate until ϕ converges. The procedure is initialized with $k^{(0)}_{j,t} = k^a_{j,t}$ and $\phi^{(0)} = 1$.

$$\phi^{(n)} = \frac{1}{NT}\sum_{j,t}\frac{k^{(n)}_{j,t}}{k^{(n-1)}_{j,t}} \quad \text{[and not in top or bottom 1\%]} \tag{16}$$

$$k^{(n+1)}_{j,1} = \phi^{(n)} k^{(n)}_{j,1} \tag{17}$$

Outliers. We exclude firms for which the economic capital stock becomes negative at any point in time. This can arise if there is a sale of capital, which exceeds current

[7] The Fixed Asset Accounts also provide depreciation rates by asset type (Equipment, Structures, Intellectual Property Products), which we do not use since the firm-level data does not include information on capital stocks or capital expenditure by asset type.

economic capital. Further, we compute the deviation between (real) accounting and economic capital stocks and discard the top 1% of that distribution. Finally, we discard firms for which we have less than 20 observations, unless they are still in the sample in the final quarter.

Evaluation. Our estimated real economic capital stock is still highly correlated with the real accounting capital stock. A simple regression has an R^2 of above 0.96 and shows that the economic capital stock is on average slightly higher (by about 4%), as expected. The investment rate (net real investment over lagged real economic capital) is highly correlated ($\rho > 0.98$) with the accounting investment rate used in Cloyne et al. (2023). A simple regression shows that on average, the economic investment rate is lower (by about 13%) than the accounting investment rate, also as expected due to the underreporting of accounting capital stocks.

D.4 Variable Construction

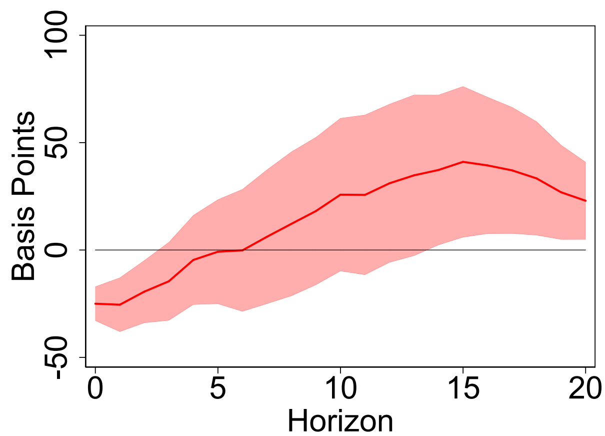
Most of our variables follow the definitions in the literature. Our baseline measure of the investment rate is $i_{jt} = \frac{CAPX_{jt} - SPPE_{jt}}{INVDEF_t \times k_{jt-1}}$, thus, real capital expenditures (CAPX) net of sales of capital (SPPE) divided by the lagged real economic capital stock, computed as described previously. To measure size, we use the log of total assets (AT).

D.5 Identification of Monetary Policy Shocks

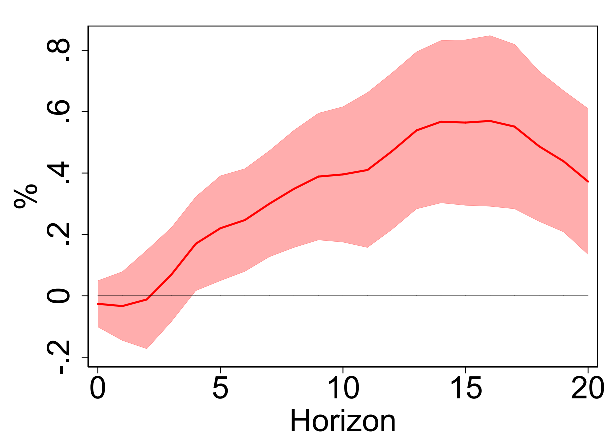
We use the monetary policy shocks implied by the proxy SVAR used in Gertler and Karadi (2015). We calculate them according to the following procedure. First, we update the data used in the Gertler and Karadi (2015) baseline SVAR. They use monthly data from 1979M7 to 2012M6. We update all time series to 2019M12. The SVAR includes (the log of) industrial production (FRED: INDPRO), (the log of) the consumer price index (FRED: CPIAUCSL), the one-year government bond rate (FRED: GS1), and the excess bond premium (Source: https://www.federalreserve.gov/econresdata/notes/feds-notes/2016/files/ebp_csv.csv, retrieved in February 2020). Moreover, we update the instrument (cumulative high-frequency FF4 surprises) to 2015M10. Then, we run the SVAR and compute the implied structural monetary policy shocks. See the appendix of Mertens and Ravn (2013) for details. Importantly, even though the instrument is only available until 2015M10, we can compute the structural monetary policy shock until 2019M12.

D.6 Effects of Monetary Policy using Aggregate Data

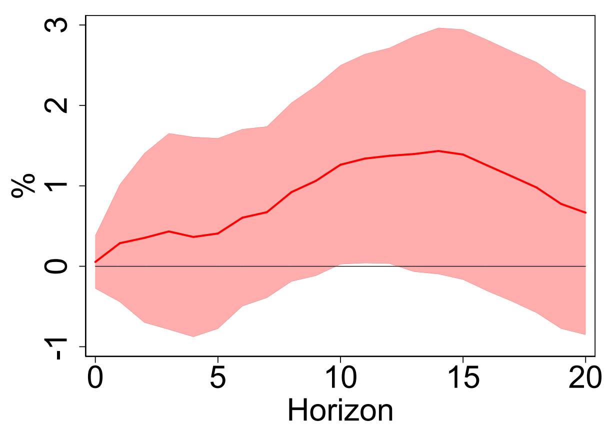
Figure D.1: Aggregate Effects of a Monetary Policy Shock



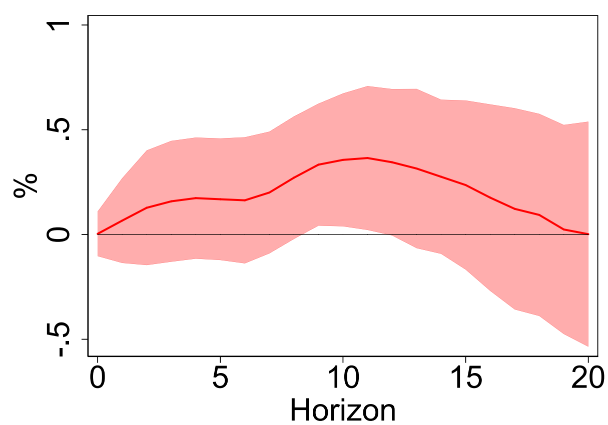
(a) 1-year Treasury yield



(b) Relative Price of Investment Goods



(c) Investment



(d) GDP

Notes: The lines represent the estimated $\hat{\beta^h}$ from separate regressions: $y_{t+h} - y_{t-1} = \alpha^h + \beta^h \epsilon_t^{MP}$ $+ \sum_{j=2}^{4} \gamma^j \mathbb{1}\{q_{t+h} = j\} + e_{t+h}$. The monetary policy shocks are scaled to reduce the 1-year Treasury yield by 25 basis points. The shaded areas are the 90% confidence intervals constructed using standard errors that are robust to heteroskedasticity and autocorrelation. All variables except for the 1-year Treasury yield are in logs.

Using time series data from FRED, we document the aggregate effects of the monetary policy shocks we utilize. Qualitatively, these are quite similar to Gertler and Karadi (2015). Panel (a) of Figure D.1 shows that a monetary policy shock decreases the 1-year Treasury yield (FRED: GS1) for roughly 4 quarters. Thereafter, it overshoots, as observed in Gertler and Karadi (2015). Panels (b) and (c) show that (real) investment (FRED: PNFI) and the relative price of capital goods (FRED: PIRIC) increase strongly. The peak effect on investment is roughly 1.4%. Panel (d) shows that real GDP (FRED: GDPC1) also increases following an expansionary shock. The peak effect is about 0.35%.

D.7 Decomposition of the Average Investment Rate

To implement the decomposition in equation (3), we construct hypothetical average investment rates that would prevail if there were no changes in the extensive margin ($\overline{i_t}^{\,int}$) or the intensive margin ($\overline{i_t}^{\,ext}$):

$$\overline{i_t}^{\,int} = \overline{\psi} i_t^s + (1 - \overline{\psi}) i_t^n, \tag{18}$$

$$\overline{i_t}^{\,ext} = \psi_t \overline{i^s} + (1 - \psi_t) \overline{i^n}. \tag{19}$$

$\overline{i_t}^{\,int}$ captures fluctuations in the average investment rate arising only from the intensive margin, because the spike rate, $\overline{\psi}$, equals its average over time. Vice versa, $\overline{i_t}^{\,ext}$ captures fluctuations in the average investment rate arising only from the extensive margin, because the conditional investment rates, $\overline{i^n}$ and $\overline{i^s}$, equal their respective averages over time.

In addition, we decompose the heterogeneous effect between young and old firms ($\frac{\partial \mathbb{E}(\overline{i_{Y,t+h}} - \overline{i_{O,t+h}})}{\partial \epsilon_t^{MP}}$) into the extensive margin ($\frac{\partial \mathbb{E}(\overline{i_{Y,t+h}}^{\,ext} - \overline{i_{O,t+h}}^{\,ext})}{\partial \epsilon_t^{MP}}$) and the intensive margin ($\frac{\partial \mathbb{E}(\overline{i_{Y,t+h}}^{\,int} - \overline{i_{O,t+h}}^{\,int})}{\partial \epsilon_t^{MP}}$).

D.8 Unconditional Fluctuations in the Average Investment Rate

The main results presented in Section 2 analyze fluctuations in the average investment rate as well as the contribution of the extensive (intensive) margin thereto *conditional* on monetary policy shocks. Our finding that the effect of monetary policy on the average investment rate is driven to a sizeable degree by the extensive margin aligns well with Gourio and Kashyap (2007), who emphasize that unconditionally, "investment spikes (the extensive margin) account for the bulk of variation in aggregate investment". In this Appendix, we complement our main results with an analysis of the *unconditional* fluctuations in the average investment rate.

To estimate the contribution of the extensive (intensive) margin to the variance of the average investment rate, we reuse the decomposition introduced in Section 2.5. Specifically, we compute the covariance of the average investment rate with the hypothetical average investment rates that would prevail if there were no changes in the extensive margin (equation 18) or the intensive margin (equation 19) and divide by the variance of the average investment rate.[8] We find that changes in the spike rate (extensive margin) account for 58% while changes in the conditional investment rates (intensive margin) account for 41% of the variance of the average investment rate.[9] Hence, we find the contribution of the extensive margin to conditional (on monetary policy shocks) and unconditional fluctuations to be of a similar magnitude. This is an important finding with implications for the investment channel of monetary policy, as discussed in Appendix E.4.

E Analysis of the Quantitative Model

E.1 Equilibrium Definition

A recursive competitive equilibrium in this model is a set of value functions $\{V_t(z,k)$, $CV_t^{exit}(z,k)$, $CV_t^a(z,k,\xi)$, $CV_t^n(z,k)\}$, policy functions $\{n_t^*(z,k)$, $k_t^*(z,k,\xi)$, $\xi_t^T(z,k)\}$, quantities $\{C_t$, Y_t, I_t^Q, K_t, $N_t\}$, prices $\{p_t$, w_t, π_t, Λ_{t+1}, $q_t\}$, and distributions $\{\mu_t(z,k)\}$ such that all agents in the economy behave optimally, the distribution of firms is consistent with decision rules, and all markets clear:

1. Investment Block: Taking all prices as given, $V_t(z,k)$, $CV_t^{exit}(z,k)$, $CV_t^a(z,k,\xi)$, and $CV_t^n(z,k)$ solve the Bellman equation with associated decision rules $n_t^*(z,k)$, $k_t^*(z,k,\xi)$, and $\xi_t^T(z,k)$.

2. Household Block: Taking prices as given, C_t and C_{t+1} satisfy the household's optimality conditions (18) and (19).

3. New Keynesian Block: The New Keynesian Phillips Curve holds. The Taylor rule holds. Taking prices a given, I_t^Q satisfies (15).

4. All markets (final good, capital, labor) clear.

[8]This is equivalent to regressing the true average investment rate on the average investment rates reflecting only one margin.

[9]The remaining 1% reflects the covariance between the spike rate and the conditional investment rates.

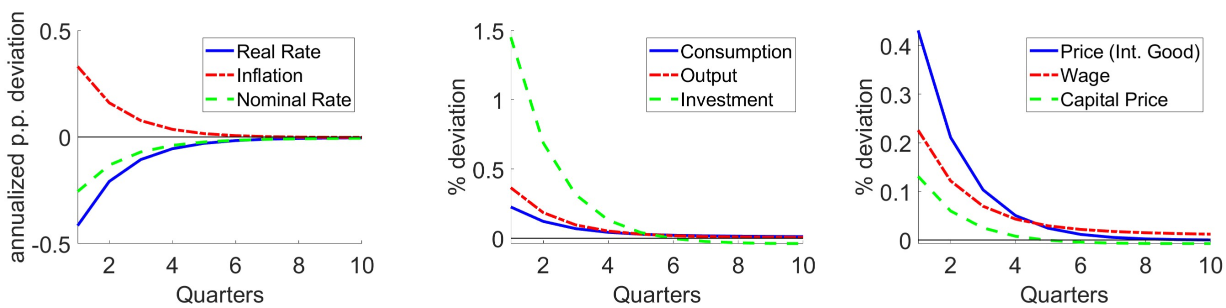
5. The distribution of firms, $\mu_t(z,k)$, evolves as implied by the decision rules $k^*(z,k,\xi)$ and $\xi_t^T(z,k)$, the exogenous process for firm-level productivity, and considering exogenous exits and entrants with capital k_0 and productivity from μ^{ent}.

E.2 Calibration

E.2.1 The Aggregate Effects of Monetary Policy Shocks

We study the effects of an unexpected expansionary monetary policy shock followed by a perfect foresight transition back to steady state.[10] Figure E.1 plots the impulse response functions of aggregates and prices, which confirm that our model produces the typical New Keynesian effects to a monetary policy shock.

Figure E.1: Aggregate Effects of an Expansionary Monetary Policy Shock



(a) Interest Rates & Inflation **(b)** Cons., Output & Investm. **(c)** Prices

Notes: This figure plots the effects of a monetary policy shock on interest rates, inflation, aggregates, and prices in the calibrated model.

E.2.2 Interest-Rate-Elasticity of Aggregate Investment

Koby and Wolf (2020) have shown that the aggregate relevance of firm heterogeneity hinges on a low elasticity of aggregate investment with respect to the interest rate. Based on quasi-experimental evidence, their preferred estimate for this elasticity at annual frequency is -5, whereas, in models without aggregate relevance of firm heterogeneity it can be as high as -500 (Khan and Thomas, 2008). In our baseline calibration, this elasticity is

[10] This approach to constructing impulse response functions to aggregate shocks follows Boppart et al. (2018). The size of the monetary shock is chosen to roughly match the peak effects on output and investment seen in the data. This implies that the nominal interest rate falls by around 25 basis points on impact.

-7.5, a value close to Koby and Wolf (2020) which implies that firm heterogeneity matters for aggregate dynamics, as studied in more detail in Section 4.3.

E.3 Discussion: Model vs. Data

Our model replicates the two empirical findings qualitatively very well, while there are some quantitative differences worth discussing. First, in the data (panel (a) of Figure A.3) the average investment rate of young firms is about 3.5 times as responsive to a monetary shock as the average investment rate of old firms. In the model (panel (a) of Figure 9), the average investment rate of young firms is only about twice as responsive. This shows that the calibrated lumpy investment model can explain a sizable share, but not all of the heterogeneous effect of monetary policy on young and old firms, suggesting the presence of additional mechanisms, such as financial acceleration.

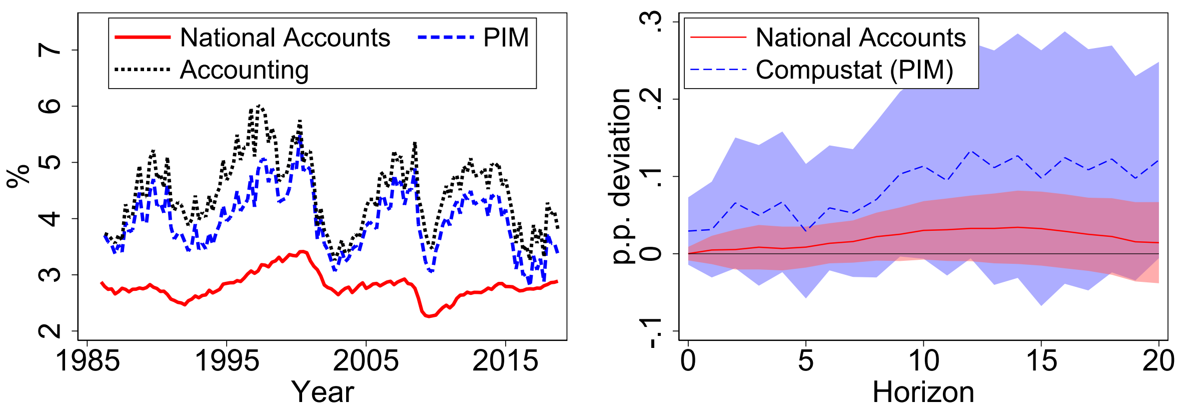
Second, while the model matches well the effect of monetary policy on aggregate investment (Figures D.1 and E.1), the effect on the average investment rate (panel (a) of Figure 4 and panel (b) of Figure 7) is substantially smaller. This primarily reflects an important discrepancy between investment data from Compustat, used to estimate this effect in the data, and investment data from national accounts, which is used to calibrate the model.[11] In particular, the aggregate investment rate in Compustat is substantially higher and more volatile than the aggregate investment rate constructed from national accounts data, as shown in panel (a) of Figure E.2.[12] In light of this, it is not surprising to find different estimates regarding the effect of monetary policy. Panel (b) of Figure E.2 plots the impulse response functions of aggregate investment rates from national accounts and from Compustat (PIM) data. While the trajectory is very similar, the magnitude differs substantially. The peak effects are about 0.03 percentage points (national accounts) and 0.13 percentage points (Compustat PIM), respectively. Since our model is calibrated to national accounts data, it quantitatively matches the former number, not the latter one.

[11] Following other studies in the literature, we use Compustat data because it offers quarterly firm-level data including information on investment rates and firm age. However, Compustat firms, being public firms, are by no means a random or representative sample of the universe of firms in the economy, giving rise to this discrepancy.

[12] Aggregate investment rates from Compustat having a higher level at least partly reflects the capital measurement issues described in Appendix D.3. The PIM addresses these issues to some extent, but the level of the investment rate remains substantially above the national-accounts investment rate. Despite the differences in the level and volatility, the investment rates are highly correlated. The aggregate investment rate from national accounts has a correlation of $\rho = 0.6$ with the "PIM" investment rate and of $\rho = 0.54$ with the "Accounting" investment rate. Both Compustat investment rates are highly correlated ($\rho = 0.95$).

Calibrating the model to national accounts data implies that the aggregate implications presented in Section 4.3 are relevant for the U.S. economy as a whole rather than a subset of the economy (publicly listed firms).

Figure E.2: Aggregate Investment Rates (NIPA & Compustat Data)



(a) Aggregate Investment Rates

(b) Effect of Monetary Policy

Notes: Panel (a) plots three quarterly aggregate investment rates. The first one is computed from national accounts data, following the procedure described in Bachmann et al. (2013). The other two are constructed from Compustat firms, reflecting two alternative ways of constructing capital. The first one uses investment and capital as computed with the perpetual inventory method ("PIM"). The second one uses investment and capital as reported in Compustat ("Accounting"). Both Compustat investment rates are seasonally adjusted using quarterly dummy variables to deal with the observation that reported investment rates are typically higher in the fourth quarter (Xu and Zwick, 2021). Panel (b) plots the estimated $\hat{\beta}^h$ from separate regressions: $y_{t+h} - y_{t-1} = \alpha^h + \beta^h \epsilon_t^{MP} + \sum_{j=2}^{4} \gamma^j \mathbb{1}\{q_{t+h} = j\} + e_{t+h}$. The monetary policy shocks are scaled to reduce the 1-year Treasury yield by 25 basis points. The shaded areas are the 90% confidence intervals constructed using standard errors that are robust to heteroskedasticity and autocorrelation.

E.4 Further Macroeconomic Shocks

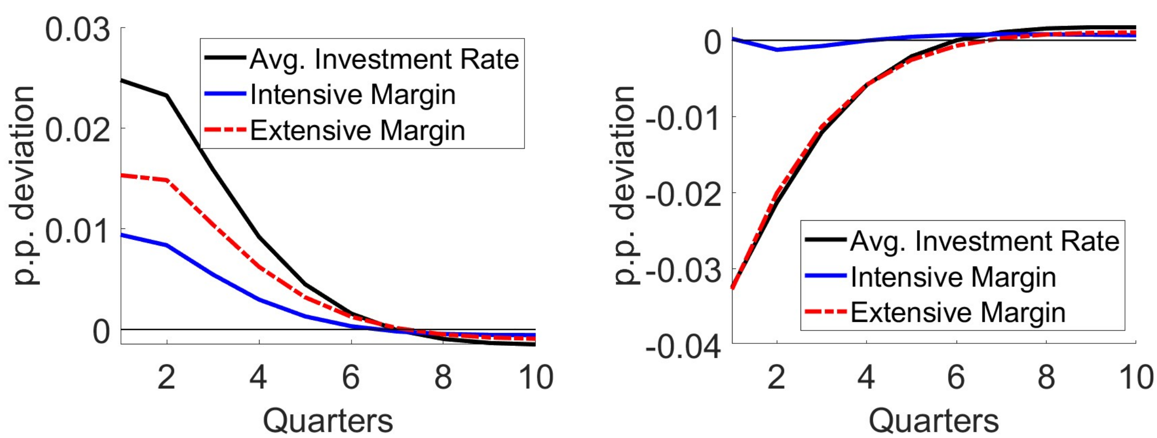
In the main text, we study the response of firm-level investment to monetary policy shocks. In this Appendix, we investigate its response to two additional macroeconomic shocks (aggregate TFP, wage-markup). The purpose is to illustrate that the contribution of the extensive margin to fluctuations in the average investment rate can differ, especially if the macroeconomic shock directly affects the capital adjustment costs.

Model Additions To introduce an aggregate TFP shock to the model, we add aggregate TFP (Z_t) to the production function (equation 4), which thereafter is $y_{jt} = Z_t z_{jt} k_{jt}^{\theta} n_{jt}^{\nu}$.

Subsequently, we consider an unexpected increase in aggregate TFP of 1%, which decays with a persistence of 0.5.

To introduce a wage-markup shock (ϵ_t^w) to the model, we add a wedge to the household's wage-Euler equation (equation 19), which thereafter is $w_t(1 - \epsilon_t^w) = \psi C_t^h$. A positive wage-markup shock (ϵ_t^w) is isomorphic to an increase in the labor disutility parameter (ψ) and requires a higher real wage for a given level of consumption.[13] Again, we subsequently consider an unexpected shock of 1%, which decays with a persistence of 0.5.

Figure E.3: Further Macroeconomic Shocks



(a) Aggregate TFP Shock

(b) Wage-Markup Shock

Notes: This figure replicates panel (b) of Figure 7 for an aggregate TFP and a wage-markup shock, respectively. See the notes of Figure 7 for more details.

Results. Figure E.3 shows and decomposes the effects of the two macroeconomic shocks on the average investment rate. As expected, higher TFP leads to higher investment, while a higher real wage leads to lower investment. Most interestingly, however, the contribution of the extensive margin differs substantially. For the aggregate TFP shock, the contribution of the extensive margin is around 60%, while for the wage-markup shock, it is close to 100%. This is because a TFP shock, like a monetary policy shock, does not affect the capital adjustment costs directly, but only indirectly via general equilibrium price effects. The wage-markup shock, in contrast, directly affects the fixed adjustment cost, because that cost is paid in units of labor. This direct effect on the fixed adjustment cost—

[13]We abstract from a microfoundation for wage-markup shocks, as provided, e.g., in Smets and Wouters (2007).

which constrains the extensive, but not the intensive margin of investment—explains the outsized role of the extensive margin.

These results highlight the value of empirically estimating the contributions of the extensive and intensive margin to investment rate fluctuations conditional on specific macroeconomic shocks, as done in Section 2 for monetary policy shocks. These estimates are informative about the modeling of capital adjustment costs, in particular, how and by which macroeconomic shocks they are affected.

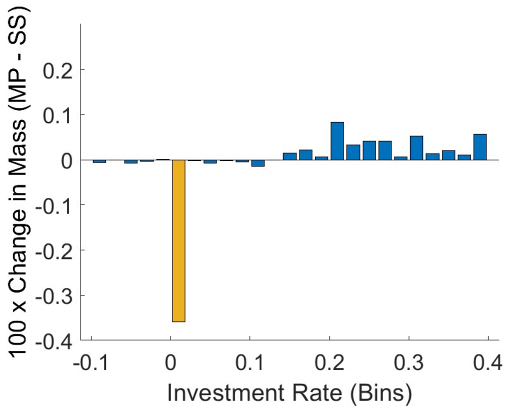
E.5 Model without Fixed Adjustment Costs

To illustrate the shortcomings of a model without an extensive margin investment decision, we study a version of our model without fixed adjustment costs ("no-FAC model"). Starting from the baseline model calibration, we set the upper bound on the fixed adjustment cost distribution, $\bar{\xi}$, to 0. All other model parameters remain unchanged. In the following, we describe which aspects of our empirical evidence the no-FAC model is at odds with.

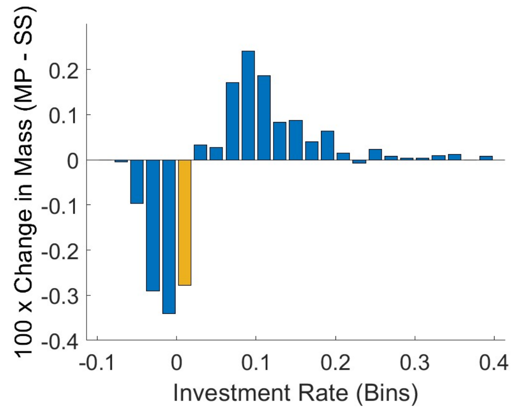
First and most importantly, the change in the distribution of investment rates after a monetary policy shock in the no-FAC model misses several aspects of the empirical evidence (Figure 1, panel (d)). As shown in panel (b) of Figure E.4, there is no outsized decrease in the share of firms in the inaction region (0% - 2%, highlighted in yellow), no meaningful increase in the share of very large investments (i.e., larger than 20%), and a counterfactual sizeable decrease in the share of negative investments. Our baseline model, replicates these aspects of the evidence, as shown in panel (a) of Figure E.4.

Second, panel (d) of Figure E.4 shows the heterogeneous effect on investment rates of young and old firms, as well as the decomposition into the extensive and intensive margin, replicating panel (b) of Figure 9. The heterogeneous effect in the no-FAC model is much smaller than in the baseline model and fully attributed to the intensive margin. Hence, the no-FAC model cannot reproduce the heterogeneous effect on young and old firms along the extensive margin.

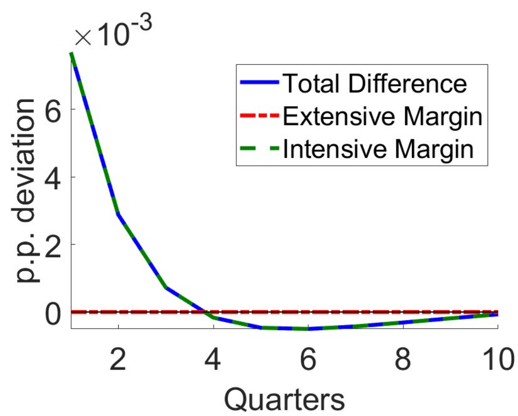
Figure E.4: Comparing Models With And Without Fixed Adjustment Costs



(a) Change in Distribution (Baseline)



(b) Change in Distribution (No FAC)



(c) Extensive vs. Intensive Margin

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